The Open University

MST121

Assignment Booklet I 2011B

Contents		Cut-off date
3	TMA MST121 01 Part 1 (covering Chapters A0 and A1)	23 February 2011
5	TMA MST121 01 Part 2 (covering Chapters A2 and A3)	23 March 2011
7	TMA MST121 02 (covering Block B)	11 May 2011
13	TMA MST121 03 (covering Block C)	06 July 2011
17	CMA MST121 41 (covering Block D)	31 August 2011

Instructions for submitting the TMAs

Send your answers to each tutor-marked assignment (TMA), together with a completed TMA form (PT3), to reach your tutor on or before the cut-off date shown above. However, you should *not* send a TMA form with *Part 2* of TMA 01 – see the submission instructions for TMA 01 on page 3.

There are instructions on how to fill in the TMA form in the *Preparatory Assignment Booklet*. Remember to fill in the correct assignment number. Remember also to allow sufficient time in the post for the TMA to reach your tutor on or before the cut-off date, and to pay the correct postage charge to cover the weight and size of the TMA and its envelope. Do not use recorded delivery. You are advised to keep a copy of your answers in case of loss in the mail, to obtain proof of posting, and to keep all your marked TMAs so you can refer to them later.

Your tutor will inform you about the address to use for submitting your TMAs. Please don't submit your TMAs directly to the University. Regrettably, the University is unable to accept TMAs submitted electronically on this module. If you have any questions about how best to prepare and submit your TMAs, please contact your tutor.

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Instructions for submitting the CMA

Submit your answers to the computer-marked assignment (CMA) online, via your StudentHome web page. There is further information on how to do this in the *Preparatory Assignment Booklet*. The CMA must be submitted by noon (UK time) on the cut-off date, but you are advised to submit it earlier, in case of computer or internet problems. Feedback will be provided via your StudentHome web page, and solutions will be provided via the MST121 website, a few days after the cut-off date.

Plagiarism

The University considers plagiarism to be a serious matter. The work that you submit must be your own and not copied from, or provided by, others. There is further information on plagiarism in the *Assessment Handbook* (accessible via your StudentHome web page).

Late information

Check the 'News' section of the MST121 website immediately before beginning each assignment, in case any late information, such as errata, has been posted. If you later suspect that an assignment question contains an error or is unsound, check the MST121 website again for errata. If you find nothing helpful there, contact your tutor about the problem.

Points to note when preparing solutions to TMA questions

- Contact your tutor if the meaning of any part of a question does not seem clear.
- Your solutions should not involve the use of Mathcad, except in those parts of questions where this is explicitly required or suggested. Your solutions should not involve the use of any other mathematical software (except OUStats for TMA 04 in *Assignment Booklet II*).
- Except where you are asked simply to 'write down' or 'state' an answer, justify your answers by showing your working and explaining your reasoning. The solutions to the activities and exercises in the Chapters and Exercise Booklets will give you an idea of how much detail is needed. You may not receive full marks for a correct final answer that is not supported by working. You may receive some marks for working even if your final answer is incorrect or your solution is incomplete.
- Whenever you perform a calculation using a numerical answer found earlier, you should use the full-calculator-precision version of the earlier answer to avoid rounding errors.
- Number all of your pages, including any computer printouts.
- Indicate in each solution the page numbers of any computer printouts associated with that solution.
- Write your name and personal identifier on each page.
- Ensure that there is space for your tutor's comments (for example, by leaving wide margins and a few blank lines).
- Use dark ink, as your answers may be photocopied.
- The marks allocated to the parts of the questions are indicated in brackets in the margin. Each TMA is marked out of 100. Your overall score for a TMA will be the sum of your marks for each question part.

Special instructions for submitting TMA MST121 01

TMA MST121 01 is in two parts. Part 1 comprises Questions 1 and 2, on Chapters A0 and A1. Part 2 comprises Questions 3–6, on Chapters A2 and A3, and follows immediately after Part 1. Part 1 is marked out of 40; the whole TMA is marked out of 100.

Please send your answers to Part 1 to your tutor, with a TMA form (PT3). Be sure to fill in the assignment number on this form as

MST121 01.

Your tutor will mark and comment on your solutions to Questions 1 and 2, and will send your script back to you directly to give you some early feedback on your work. The TMA form relates to the whole assignment, so it will not be returned to you at that stage, but will be retained by your tutor so that your marks for Part 2 of the assignment may be entered on it. The form will then be sent to you, with your marked solutions to Part 2, via Walton Hall so that your marks can be recorded.

Question 1 – 20 marks

You should be able to answer this question after studying Chapter A0.

(a) Solve the following pair of simultaneous equations algebraically. Any non-integer solutions should be given as fractions in their simplest form.

$$2a + 7b = -1$$
$$3a + 4b = 5$$ [4]

(b) Rearrange the following equation to find x in terms of a and b, in simplest form. (Assume that $2a \neq b$.)

$$2a(x - 2a) = b(x - b)$$ [4]

(c) Solve the following equation for x.

$$\frac{x-1}{x} - \frac{4}{x+3} = 0$$ [4]

In parts (d) and (e) you are asked to use Mathcad. You should provide printouts of your work for parts (d) and (e), which may be on a single page.

(d) Create a new Mathcad worksheet, and define a variable p to have value 1.25. Type in the expression below, and evaluate it for the given value of p.

$$3p^3 - 8p^2 - 5p + 17$$

Now choose **Result...** from the **Format** menu and alter the Number Format to display your answer correct to six decimal places. [4]

(e) Enter the expression below into a Mathcad worksheet, and use Mathcad to factorise it.

$$48x^3 + 28x^2 - 188x + 105$$ [4]

Question 2 – 20 marks

You should be able to answer this question after studying Chapter A1.

(a) Consider the following recurrence system, which represents an arithmetic sequence:

$$u_1 = 2, \quad u_{n+1} = u_n - 0.7 \quad (n = 1, 2, 3, \ldots).$$

(i) Write down the first five terms of the sequence. [*2*]

(ii) Find a closed form for the sequence. [*2*]

(b) Consider the following sequence:

$$2.5, \quad 2.0, \quad 1.6, \quad 1.28, \quad \ldots.$$

(i) State what type of sequence this is, and give the fifth number in the sequence, explaining your reasoning. [*3*]

(ii) Write down a recurrence system that describes this sequence. (Denote the sequence by u_n, and its first term by u_1.) [*2*]

(iii) Find a closed form for the sequence. [*2*]

(iv) Use the closed form that you found in part (b)(iii) to find the 15th term of the sequence, giving your answer to five significant figures. [*2*]

(c) Consider the following linear recurrence sequence:

$$x_1 = 10, \quad x_{n+1} = 0.6x_n + 5 \quad (n = 1, 2, 3, \ldots).$$

(i) Find a closed form for this sequence. [*4*]

(ii) Use the closed form to find the 11th term of the sequence, correct to four decimal places. [*1*]

(iii) Describe the long-term behaviour of the sequence, briefly justifying your answer. [*2*]

This part of the TMA comprises Questions 3–6, and covers Chapters A2 and A3.

Please remember that you should NOT submit a second PT3 form with this part of the assignment.

Question 3 – 20 marks

You should be able to answer this question after studying Chapter A2.

This question concerns the three points $A(-1, 5)$, $B(-7, 7)$ and $C(-1, 1)$, and the circle that passes through them.

(a) (i) Find the slope of the line that passes through A and B. [2]

(ii) Find the coordinates of the midpoint of the line segment AB. [2]

(iii) Use your answers from parts (a)(i) and (a)(ii) to find the equation of the perpendicular bisector of AB. [2]

(iv) Show, by eliminating t, that the line corresponding to the parametric equations

$$x = t - 4, \quad y = 3t + 6$$

is the same line as in part (a)(iii) above. [2]

(b) Find the equation of the perpendicular bisector of the line segment BC. [4]

(c) (i) From your answers to parts (a)(iii) and (b), or otherwise, find the centre of the circle that passes through the points A, B and C. [4]

(ii) Find the radius of this circle, giving your answer correct to four decimal places. [2]

(iii) Write down the equation of the circle. There is no need to simplify your answer. [2]

(As a check, note that the coordinates of each of the points A, B and C should satisfy this equation.)

Question 4 – 10 marks

You should be able to answer this question after studying Chapter A2.

(a) The equation

$$x^2 + y^2 - 2x + 10y + 9 = 0$$

represents a circle. Find its centre and radius. [6]

(b) Find the coordinates of any points at which the circle in part (a) intersects the line $3y = x - 5$. [4]

Question 5 – 20 marks

You should be able to answer this question after studying Chapter A3.

(a) This part of the question concerns the graph of the function

$$f(x) = -2(x+3)^2 + 8.$$

(i) Explain how the parabola that is the graph of f can be obtained from the graph of $y = x^2$ by using appropriate translations and scalings. [*4*]

(ii) Write down the coordinates of the vertex of the parabola. [*1*]

(iii) Find the x-intercepts and y-intercept of the parabola. [*3*]

(iv) Sketch the parabola, marking on your sketch the coordinates of the vertex and of the points at which the graph crosses the axes. [*3*]

This should be your own sketch, not a Mathcad printout.

(v) What is the image set of the function f? Express your answer in interval notation. [*2*]

(vi) Explain why the function f as defined above does not have an inverse. [*1*]

(b) This part of the question concerns the function

$$g(x) = -2(x+3)^2 + 8 \quad (-5 \leq x \leq -3).$$

(The function g has the same rule as the function f in part (a), but a smaller domain.)

(i) Specify the domain and image set of the inverse function g^{-1}, and find its rule. [*5*]

(ii) Sketch the graph of $y = g^{-1}(x)$. [*1*]

Question 6 – 10 marks

You should be able to answer this question after studying Chapter A3.

(a) Solve the equation $8 = 3^x$ by applying ln (the natural logarithm function, $\log_e$) to both sides. Give your answer to five decimal places. [*3*]

You should provide printouts of your solutions to parts (b) and (c).

(b) Check your answer to part (a) by using a Mathcad 'solve block' to solve the equation $8 = 3^x$ as it stands, obtaining a numerical solution to five decimal places using the methods covered in Mathcad file 121A3-02. [*3*]

(c) (i) Use Mathcad to plot the graph of the function

$$f(x) = 8 - 3^x \quad (-1 \leq x \leq 3).$$

(You might like to start from Mathcad file 121A3-04.) [*3*]

(ii) Add to your graph the line $y = 0$, and indicate on this line the location of your solution to part (a). [*1*]

This assignment covers Block B. It has six questions.

Question 1 – 20 marks

You should be able to answer this question after studying Chapter B1.

A troop of howler monkeys was introduced onto an island on 1 July 1965, and the approximate size of the population was estimated on 1 July in each subsequent year. The size of the initial population was 50, and it had grown to approximately 65 after one year. Some years later, the size of the population was found to be approximately 550, and the following year it had grown to approximately 660. Assume that the behaviour of this population satisfies the logistic model.

(a) Show that the annual proportionate growth rate for the population of size 50 was approximately 0.3, and that the annual proportionate growth rate for the population of size 550 was approximately 0.2. [*3*]

(b) Find the corresponding values of the annual proportionate growth rate for low population levels, r, and the equilibrium population size, E. [*7*]

You should use Mathcad file 121B1-01 in parts (c) and (d) below. Provide a printout of page 2 of the amended worksheet, showing your work.

(c) By adjusting values of variables in a copy of the worksheet in Mathcad file 121B1-01, produce a Mathcad graph showing the behaviour of the population over the first 40 years, as predicted by the model. [*4*]

(d) According to the model, how many years does it take until the size of the population on 1 July is within 4% of its equilibrium level? [*4*]

(e) Describe the long-term behaviour of the population as predicted by the model, justifying your answer without reference to Mathcad. [*2*]

Question 2 – 15 marks

You should be able to answer this question after studying Chapter B1.

(a) (i) Find the sum of the integers from 37 to 112 inclusive. [*3*]

(ii) Hence find the value of $\sum_{i=37}^{112}(9+6i)$. [*3*]

(b) For each of the sequences below, decide whether it converges and, if it does, state its limit. Justify your answers briefly.

(i) $a_n = \dfrac{6n^4+2n^3}{7n-11n^4} \quad (n=1,2,3,\ldots)$ [*3*]

(ii) $b_n = \dfrac{3-3^n}{7(0.9)^n+8} \quad (n=1,2,3,\ldots)$ [*3*]

(c) Find the fraction equivalent to the infinite decimal

$$0.752\,475\,247\,524\ldots.$$ [*3*]

Question 3 – 15 marks

You should be able to answer this question after studying Chapter B2.

This question concerns the following matrices.

$$\mathbf{A} = \begin{pmatrix} 9 & -12 \\ 6 & -8 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 4 & -7 \\ 8 & 5 \\ -3 & 2 \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} 4 & 2 \\ 3 & 5 \end{pmatrix}$$

(a) Evaluate each of the following, where possible. (Give sufficient details of your working to make it clear that you have not needed to use Mathcad.) Where evaluation is not possible, explain why not.

(i) $3\mathbf{A} - 2\mathbf{B}$

(ii) $2\mathbf{A} + 5\mathbf{C}$

(iii) $\mathbf{AB}$

(iv) $\mathbf{BA}$

(v) $\mathbf{A}^{-1}$

(vi) $\mathbf{C}^{-1}$ [*11*]

(b) Use matrices to solve the following system of linear equations.

$$4x + 2y = 14$$
$$3x + 5y = 7$$ [*4*]

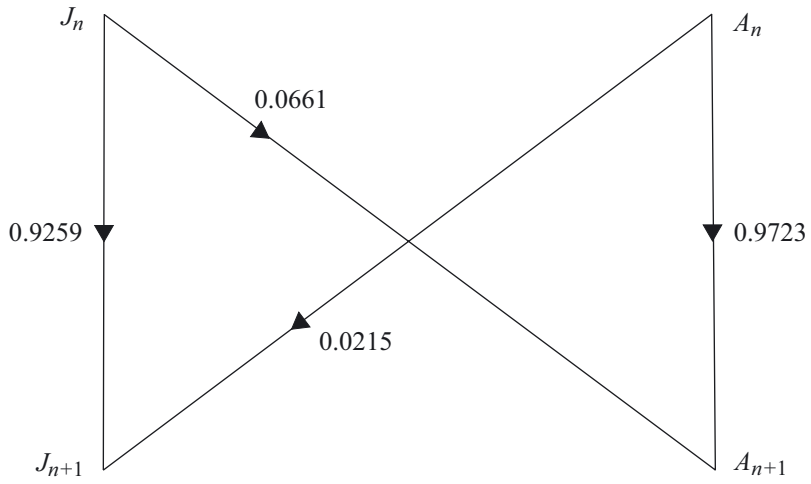
Question 4 – 20 marks

You should be able to answer this question after studying Chapter B2.

The following table gives data for a fictitious country in the year 1930.

	Juveniles (aged under 15 years)	Adults (aged 15 years and over)
Subpopulation size (in millions)	2.14	6.82
Birth rate	0	0.0215
Death rate	0.008	0.0277

The diagram below shows a network model of the changes in the sizes of the subpopulations from year to year. Here J_n and A_n denote the sizes (in millions) of the subpopulations of juveniles and adults, respectively, n years after the census date in 1930. It has been assumed that, in any year, $\frac{1}{15}$ of the juveniles become adults.



(a) Explain why the number labelling the connection J_n to A_{n+1} is 0.0661, and the number labelling the connection J_n to J_{n+1} is 0.9259. [4]

(b) The network model above can be written as a matrix equation of the form

$$\begin{pmatrix} J_{n+1} \\ A_{n+1} \end{pmatrix} = \mathbf{M} \begin{pmatrix} J_n \\ A_n \end{pmatrix},$$

where $\mathbf{M}$ is a 2×2 matrix. Write down the matrix $\mathbf{M}$. [2]

For parts (c) and (d) you will need to use Mathcad file 121B2-02. For each of these parts, provide a printout of pages 2–4 of the amended worksheet, showing your work.

(c) (i) Edit the matrix $\mathbf{M}$, and the vector whose entries are the initial subpopulation sizes J_0 and A_0, in a copy of the worksheet in Mathcad file 121B2-02, so that the worksheet shows the predicted changes in population for the country considered in this question. Set $N = 100$, so that the long-term behaviour can be seen. [2]

(ii) Describe what happens to the size of the adult subpopulation, the size of the total population and the ratio of successive total populations over the 100-year period of prediction. Your answers should include both descriptions of behaviour and numerical information. Any values of the ratio of successive total populations should be given to five decimal places. [*4*]

(iii) It is estimated that the adult subpopulation can comfortably support the juvenile subpopulation only if the size of the juvenile subpopulation is at most 4/9 of the size of the adult subpopulation. Determine whether this is achieved by the end of the 100-year period of prediction. [*2*]

(d) Suppose now that there is an annual net immigration to the country of 0.0007 million juveniles and 0.0054 million adults. The matrix equation for the revised model, taking account of this immigration, is

$$\begin{pmatrix} J_{n+1} \\ A_{n+1} \end{pmatrix} = \mathbf{M} \begin{pmatrix} J_n \\ A_n \end{pmatrix} + \begin{pmatrix} 0.0007 \\ 0.0054 \end{pmatrix},$$

where $\mathbf{M}$ is the same matrix as in part (b).

(i) The matrix equation from part (b) appears as a Mathcad formula on page 2 of the worksheet. Edit this formula to make it the equation for the revised model. (To do this, click anywhere on the right-hand side of the formula, press the right-arrow key repeatedly until the vertical blue editing line is positioned just to the right of the formula, and type '+'. Then use the 'Matrix' toolbar to create a 2×1 matrix, and enter the appropriate values in the placeholders.) [*2*]

(ii) Describe what happens to the size of the total population and the ratio of successive total populations over the 100-year period of prediction. Your answers should include both descriptions of behaviour and numerical information. Any values of the ratio of successive total populations should be given to five decimal places. [*2*]

(iii) Repeat part (c)(iii) for the revised model. [*2*]

Question 5 – 15 marks

You should be able to answer this question after studying Chapter B3.

Each of parts (b), (c) and (d) requires you to use numerical answers found in earlier parts of the question. Make sure that you use the full-calculator-accuracy versions of the earlier answers, to avoid rounding errors.

You may find it helpful to draw a diagram for this question.

A hawk has a flight speed in still air of $10.4\,\mathrm{m\,s^{-1}}$. It is pointed in the direction N77°W but flies in a wind of speed $8.3\,\mathrm{m\,s^{-1}}$ *from* the direction N48°E. Take $\mathbf{i}$ to be $1\,\mathrm{m\,s^{-1}}$ due east and $\mathbf{j}$ to be $1\,\mathrm{m\,s^{-1}}$ due north. Also, take

$\mathbf{v}_\mathrm{h}$ to be the velocity of the hawk in still air,
$\mathbf{v}_\mathrm{w}$ to be the velocity of the wind,
$\mathbf{v}$ to be the resultant velocity of the hawk.

(a) Express each of the vectors $\mathbf{v}_\mathrm{h}$ and $\mathbf{v}_\mathrm{w}$ in component form, giving the components to four decimal places. [5]

(b) Hence show that the resultant velocity $\mathbf{v}$ of the hawk is given in component form approximately by

$$\mathbf{v} = -16.3016\,\mathbf{i} - 3.2143\,\mathbf{j}.$$ [2]

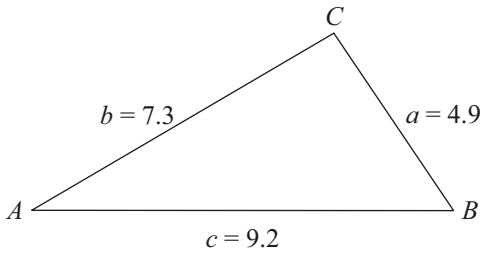
(c) Find the overall speed $|\mathbf{v}|$ of the hawk (in $\mathrm{m\,s^{-1}}$ to one decimal place), and its direction of travel, as a bearing (with the angle in degrees to one decimal place). [5]

(d) The hawk begins its flight from a point on the north bank of a river that flows due east and is 48 metres wide. How long does it take the hawk to cross the river, and what distance has it travelled in that time? Give your answers in seconds and metres, respectively, to three significant figures. [3]

Question 6 – 15 marks

You should be able to answer this question after studying Chapter B3.

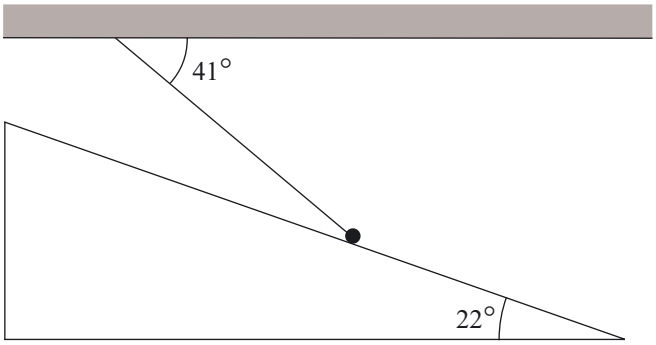
(a) The triangle ABC, labelled according to the convention shown in Figure 3.1(a) on page 30 of Chapter B3, has side lengths $a = 4.9$, $b = 7.3$ and $c = 9.2$.



(i) Use the Cosine Rule to find the angle A, in degrees to one decimal place. [2]

(ii) Hence find the angles B and C, in degrees to one decimal place. [3]

(b) A ball of mass 1.7 kg rests on a ramp that makes an angle of 22° with the horizontal. It is held in place by a rope that makes an angle of 41° with the horizontal, as shown in the diagram. Assume that the only forces acting on the ball are its weight, the tension in the rope and the normal reaction from the ramp. Take the magnitude of the acceleration due to gravity to be $g = 10\,\mathrm{m\,s^{-2}}$.



(i) Draw a force diagram for the forces acting on the ball, giving the sizes of the angles between the forces, and defining any symbols that you use to denote the forces. [3]

(ii) Draw a corresponding triangle of forces, indicating the sizes of the angles. [3]

(iii) Use the triangle of forces to find the magnitudes of the tension in the rope and the normal reaction from the ramp, in newtons to one decimal place. [4]

This assignment covers Block C. It has six questions.

Question 1 – 20 marks

You should be able to answer this question after studying Chapter C1.

This question concerns the function

$$f(x) = x^3 - 15x^2 + 48x + 29.$$

(a) Find the stationary points of this function. [6]

(b) (i) Using the strategy to apply the First Derivative Test, classify the left-hand stationary point found in part (a). [4]

(ii) Using the Second Derivative Test, classify the right-hand stationary point found in part (a). [3]

(c) Find the y-coordinate of each of the stationary points on the graph of the function $f(x)$, and also evaluate $f(0)$. [3]

(d) Hence draw a rough sketch of the graph of the function $f(x)$. [4]

Question 2 – 15 marks

You should be able to answer this question after studying Chapter C1.

In each of the following parts, you should simplify your answers where it is appropriate to do so.

(a) (i) Write down the derivative of each of the functions

$$f(x) = e^{-3x} \quad \text{and} \quad g(x) = \ln(6x) \quad (x > 0).$$ [2]

(ii) Hence, by using the Product Rule, differentiate the function

$$k(x) = e^{-3x}\ln(6x) \quad (x > 0).$$ [2]

(b) (i) Write down the derivative of each of the functions

$$f(t) = 2 - t^2 \quad \text{and} \quad g(t) = \cos(4t).$$ [2]

(ii) Hence, by using the Quotient Rule, differentiate the function

$$k(t) = \frac{2 - t^2}{\cos(4t)} \quad (-\tfrac{1}{8}\pi < t < \tfrac{1}{8}\pi).$$ [3]

(c) (i) Write down the derivative of the function

$$f(x) = \frac{5}{x} \quad (x > 0).$$ [1]

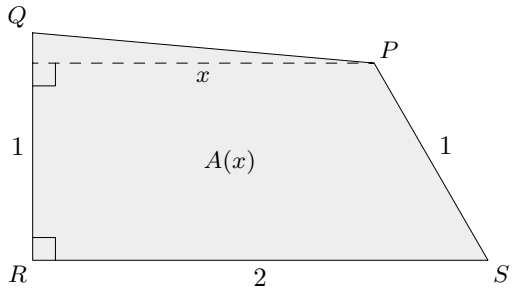
(ii) Hence, by using the Composite Rule, differentiate the function

$$k(x) = \sin\left(\frac{5}{x}\right) \quad (x > 0).$$ [5]

Question 3 – 10 marks

You should be able to answer this question after studying Chapter C1.

In the quadrilateral $PQRS$ shown below, the sides QR and PS each have length 1 metre, the side RS has length 2 metres, and the angle at R is a right angle. The point P is a perpendicular distance x metres from QR. The value of x is between 1 and 3. (The quadrilateral described cannot exist for other values of x.)



The total area $A(x)\,\mathrm{m}^2$ of the quadrilateral is given by

$$A(x) = \tfrac{1}{2}x + \sqrt{(x-1)(3-x)} \quad (1 < x < 3).$$

(You are *not* asked to derive this formula.)

For parts (a) and (b) (and for part (c), if you use Mathcad there) you should provide a printout annotated with enough explanation to make it clear what you have done.

Note: If you define x to be a range variable in part (a) and wish to use x in a symbolic calculation in part (b), then you will need to insert the definition $x := x$ between the two parts in your worksheet. (*For more details, see the bottom of page 49 in* Guide to Mathcad.)

(a) Use Mathcad to obtain the graph of the function $A(x)$. [2]

(b) This part of the question requires the use of Mathcad in each sub-part.

(i) By using the differentiation facility, and if you wish the symbolic keyword 'simplify', find an expression for the derivative $A'(x)$. [2]

(ii) By either applying a solve block or solving symbolically, find a value of x for which $A'(x) = 0$. [2]

(iii) Verify, by the Second Derivative Test, that this value of x corresponds to a local maximum of $A(x)$. (It should be apparent from the graph obtained in part (a) that this is also an overall maximum within the domain of $A(x)$.) [2]

(c) Using Mathcad, or otherwise, calculate the maximum possible area of the quadrilateral, according to the model. [2]

Question 4 – 25 marks

You should be able to answer this question after studying Chapter C2.

(a) Find the indefinite integrals of the following functions.

(i) $f(t) = 14\sin(7t) + 5e^{-15t}$ [3]

(ii) $g(x) = \dfrac{8 + 39x^3}{x} \quad (x > 0)$ [4]

(iii) $h(u) = \sin^2\left(\frac{1}{8}u\right)$ [6]

(b) Evaluate $\displaystyle\int_2^6 x(8 - 5x^2)\,dx$. [6]

(c) (i) Write down a definite integral that will give the value of the area under the curve $y = x^3\cos(6x)$ between $x = \frac{1}{4}\pi$ and $x = \frac{1}{3}\pi$. [2]

(The expression $x^3\cos(6x)$ takes no negative values for $\frac{1}{4}\pi \le x \le \frac{1}{3}\pi$. You are *not* asked to evaluate the integral by hand.)

Provide a printout of your working for part (c)(ii).

(ii) Use Mathcad to find the area described in part (c)(i), giving your answer correct to four significant figures. [4]

Question 5 – 10 marks

You should be able to answer this question after studying Chapter C2.

A rocket is modelled by a particle that moves along a vertical line. From launch, the rocket rises until its motor cuts out after 9 seconds. At this time it has reached a height of 380 metres above the launch pad and attained an upward velocity of $40\,\mathrm{m\,s^{-1}}$. From this time on, the rocket has a constant upward acceleration of $-10\,\mathrm{m\,s^{-2}}$ (due to the effect of gravity alone).

Choose the s-axis (for the position of the particle that represents the rocket) to point upwards, with origin at the launch pad. Take $t = 0$ to be the time when the rocket motor cuts out.

(a) What is the maximum height (above the launch pad) reached by the rocket? [4]

(b) How long (from launch) does the rocket take to reach this maximum height? [2]

(c) After how long (from launch) does the rocket crash on to the launch pad? Give your answer in seconds correct to one decimal place. [4]

Question 6 – 20 marks

You should be able to answer this question after studying Chapter C3.

(a) Solve the initial-value problem

$$\frac{dy}{dx} = \frac{\cos(3x)}{2 - \sin(3x)}, \quad y = 2 \text{ when } x = 0.$$

(You may find equation (2.4) in Chapter C2 helpful when integrating.) [*6*]

(b) (i) Using equation (2.3) in Chapter C2, show that

$$\int \frac{e^x + e^{-2x}}{(2e^x - e^{-2x})^{3/2}}\, dx = -(2e^x - e^{-2x})^{-1/2} + c,$$

where c is an arbitrary constant. [*2*]

(ii) Hence find, in implicit form, the general solution of the differential equation

$$\frac{dy}{dx} = \frac{2y^{5/4}\left(e^x + e^{-2x}\right)}{(2e^x - e^{-2x})^{3/2}} \quad (y > 0).$$ [*5*]

(iii) Find the corresponding particular solution (in implicit form) that satisfies the initial condition $y = 16$ when $x = 0$. [*3*]

(iv) Find the explicit form of this particular solution. [*2*]

(v) What is the value of y given by this particular solution when $x = \frac{1}{2}$? Give your answer to four significant figures. [*2*]

This assignment covers Block D. It has 24 questions.

Questions 1 to 9 are on Chapter D1.

Questions 1 to 4

Two fair dice, one six-sided and one tetrahedral, are rolled.

1 Choose the option that gives the probability that the numbers obtained on the two dice add up to 7.

2 Choose the option that gives the probability that the numbers obtained on the two dice add up to 6 or more.

Options for Questions 1 and 2

A	$\frac{1}{7}$	**B**	$\frac{1}{6}$	**C**	$\frac{1}{3}$	**D**	$\frac{5}{12}$
E	$\frac{7}{12}$	**F**	$\frac{15}{24}$	**G**	$\frac{3}{4}$	**H**	$\frac{5}{6}$

3 Choose the option that gives the probability that the number on the six-sided die is 5 or 6 and the number on the tetrahedral die is odd.

4 Choose the option that gives the probability that the number 3 is obtained on at least one of the dice.

Options for Questions 3 and 4

A	$\frac{1}{8}$	**B**	$\frac{1}{6}$	**C**	$\frac{5}{24}$	**D**	$\frac{1}{4}$
E	$\frac{1}{3}$	**F**	$\frac{3}{8}$	**G**	$\frac{5}{12}$	**H**	$\frac{11}{24}$

Questions 5 and 6

During one day in a particular hospital, the probability that a birth was that of a boy was $\frac{26}{41}$.

5 Choose the option that is closest to the probability that the first boy born on that day was the fifth child born.

6 Choose the option that is closest to the probability that there were at least four girls born before the first boy was born.

Options for Questions 5 and 6

A	0.0114	**B**	0.0179	**C**	0.0592	**D**	0.1025
E	0.1853	**F**	0.6341	**G**	0.7	**H**	0.9615

Question 7

A popular pastime in a particular club is flipping beer mats balanced on the edge of a table and trying to catch them before they land back on the table. Ann is successful two times out of five in the long run. Choose the option that is closest to the number of times, on average, that she has to flip a beer mat in order to catch it.

Options

A 0.4 **B** 0.5 **C** 1 **D** 1.2 **E** 2 **F** 2.5

Question 8

A game uses a pack of five cards numbered 1, 2, 3, 4, 5. In each round the cards are shuffled, the top card is turned face up, and the number is recorded. This card is then returned to the pack. Choose the option that gives, on average, the number of rounds to two decimal places that would be required to obtain four different numbers.

Options

A 5.42 **B** 6.42 **C** 7.42

D 8 **E** 8.42 **F** 10.42

Question 9

Assuming that a birth is equally likely to occur on any day of the week, choose the option that is closest to the probability that, in a family of four children, at least two of the children were born on the same day of the week.

Options

A 0.02 **B** 0.10 **C** 0.35

D 0.45 **E** 0.55 **F** 0.65

Questions 10 to 13 are on Chapter D2.

Question 10

A sample of concentrations of transferrin receptor for six women with laboratory evidence of overt iron-deficiency anaemia yielded the following data, in ppm.

12.1 5.3 7.7 5.4 5.9 13.2

Choose the option that is closest to the sample standard deviation.

Options

A 3.19 **B** 3.52 **C** 8.30

D 12.23 **E** 24.96 **F** 27.34

Questions 11 to 13

You should use OUStats for these questions.

The weights (in kg) of people in a sample selected at random from a particular population are normally distributed with mean 73.15 and standard deviation 6.6.

11 Choose the option that is closest to the weight above which approximately 25% of the weights of people from the population will lie.

Options for Question 11

A 55.20 **B** 55.87 **C** 56.63

D 60.24 **E** 68.70 **F** 77.60

12 Choose the option that is closest to a range of values, symmetric about the mean, within which approximately 95% of the weights of people will lie.

Options for Question 12

A $(60.2, 86.1)$ **B** $(60.6, 85.7)$ **C** $(62.3, 84.0)$

D $(66.9, 88.4)$ **E** $(62.6, 83.7)$ **F** $(61.6, 84.7)$

13 Choose the option that is closest to the percentage of people with weights below 90 kg.

Options for Question 13

A 1% **B** 10% **C** 19%

D 81% **E** 90% **F** 99%

Questions 14 to 17 are on Chapter D3.

Questions 14 to 16

The distribution of the weights of the contents of boxes of a certain breakfast cereal, labelled as containing 450 g, has mean 452.1 g and standard deviation 6.5 g.

14 Choose the option that is closest to the standard error of the mean contents (in grams) for samples of 30 cereal boxes.

Options for Question 14

A 0.22 **B** 0.32 **C** 1.19

D 1.21 **E** 1.30 **F** 2.5

You should use OUStats for Questions 15 and 16.

The sampling distribution of the mean weight of contents, for samples of 35 cereal boxes, has mean 452.1 g and standard deviation 1.1 g.

15 Choose the option that is closest to the probability that the mean weight of the contents of a sample of 35 cereal boxes will be less than 450 g.

Options for Question 15

A	0.03	**B**	0.17	**C**	0.31
D	0.53	**E**	0.79	**F**	0.97

16 Choose the option that gives a range of values, symmetric about the mean, within which the mean weight of contents of approximately 90% of samples of 35 cereal boxes will lie.

Options for Question 16

A	$(449.2, 455.0)$	**B**	$(449.4, 454.8)$	**C**	$(449.9, 454.3)$
D	$(450.3, 453.9)$	**E**	$(451.0, 453.2)$	**F**	$(451.4, 452.8)$

Question 17

The mean monthly expenditure on petrol per household in Milton Keynes is estimated by selecting a random sample of 36 households. The sample mean is £186.25, and the sample standard deviation is £47.40.

Choose the option that gives an approximate 95% confidence interval for the mean monthly expenditure on petrol per household in Milton Keynes.

Options

A	$(171.24, 201.26)$	**B**	$(173.29, 199.21)$	**C**	$(175.19, 197.31)$
D	$(170.77, 201.73)$	**E**	$(173.61, 198.89)$	**F**	$(174.80, 197.70)$

Questions 18 to 24 are on Chapter D4.

Questions 18 to 20

The salaries of employees (in £) in a certain small company are as follows.

9829	12 554	30 802	23 664	17 582
41 541	29 263	27 486	45 859	14 291

18 Choose the option that is the median salary in £ in the company.

Options for Question 18

A	9829	**B**	23 664	**C**	25 287
D	25 575	**E**	27 486	**F**	45 859

19 Choose the TWO options that give the lower and upper quartiles for the salaries in the company.

Options for Question 19

A	9829	**B**	14 291	**C**	17 582
D	29 263	**E**	30 802	**F**	45 859

20 Choose the option that gives the range of the salaries in the company.

Options for Question 20

A	9829	**B**	18 015	**C**	25 287
D	25 575	**E**	36 030	**F**	45 859

Questions 21 and 22

A small supermarket wishes to test the effectiveness of two types of coupon for a day. The amount (in £) spent by each customer using a coupon is recorded.

A summary of the results is given in the following table.

Coupon	Sample size	Amount (in £) spent during the day: Sample mean	Amount (in £) spent during the day: Sample standard deviation
Type I	38	26.27	4.37
Type II	32	24.05	9.4

21 Choose the option that is closest to the estimated standard error (ESE) of the difference between the sample means for the first and second types of coupon.

Options for Question 21

A 0.11 **B** 0.64 **C** 1.81

D 3.26 **E** 6.02 **F** 10.37

22 Choose the option that is closest to the magnitude of the test statistic z that would be used to carry out a two-sample z-test to determine whether there is a difference between the mean amount spent with the two types of coupon.

Options for Question 22

A 0 **B** 0.06 **C** 1.11

D 1.23 **E** 2.22 **F** 27.80

Questions 23 and 24

A production manager in a certain engineering company uses a regression line to model the relationship between production volume and production cost. The regression line obtained using the recorded data has the equation

$$y = 4.4 + 3.2x,$$

where y is the production cost (in £000s), and x is the units produced (in 000s).

23 Choose the option that gives the estimated production cost (in £000s) for producing 5500 units.

Options for Question 23

A	17.60	**B**	22.00	**C**	27.40
D	5500	**E**	17 604.40	**F**	17 600 004.40

24 Choose the option that gives the estimated increase in the production cost (in £000s) due to a production increase from 4000 to 5000 units.

Options for Question 24

A	3.2	**B**	4.4	**C**	10
D	1000	**E**	3200	**F**	4400